Preschoolers' errors in length measurement: a case study

STAMATINA KOLOVOU[1], KONSTANTINOS LAVIDAS[1], ANASTASIA MISIRLI[1], VASSILIS KOMIS[1], STAMATIOS PAPADAKIS[2]

[1]Department of Educational Sciences
and Early Childhood Education
University of Patras
Greece
inakolovou98@gmail.com
lavidas@upatras.gr
amisirli@upatras.gr
komis@upatras.gr

[2]Department of Preschool Education
University of Crete
Greece
stpapadakis@uoc.gr

ABSTRACT
Measurement is an important part of mathematics in preschool education, covering many essential concepts for young children. However, research in this area is still limited. When learning about concepts like length, children often make errors due to their thinking strategies. Understanding these errors is crucial for teachers to improve their teaching. This study focuses on the errors preschool children make when using both standard and nonstandard tools for direct and indirect measurement. The research involved 20 preschoolers and included a teaching intervention that combined traditional methods with four digital educational games. The results showed that children often made common errors, such as: leaving gaps between nonstandard units when measuring, placing rulers incorrectly, and misunderstanding the numeric intervals on rulers. These findings emphasize the need for teachers to address such errors to improve children's understanding of measurement. The study offers valuable insights into children's challenges when learning about measurement. It also provides practical recommendations for teachers and policymakers to create effective teaching strategies and interventions.

KEYWORDS
Kindergarten, geometry, comparisons, standard, nonstandard measures, digital educational games

RÉSUMÉ
La mesure est une partie importante des mathématiques en éducation préscolaire, couvrant de nombreux concepts essentiels pour les jeunes enfants. Cependant, la recherche dans ce domaine reste limitée. Lorsqu'ils apprennent des concepts comme la longueur, les enfants commettent souvent des erreurs à cause de leurs stratégies de réflexion. Comprendre ces erreurs est crucial pour que les enseignants améliorent leur enseignement. Cette étude se concentre sur les erreurs que commettent les enfants d'âge préscolaire lorsqu'ils utilisent à la fois des outils standards et non standards pour la mesure directe et indirecte. La recherche a impliqué 20 enfants d'âge

préscolaire et comprenait une intervention pédagogique combinant méthodes traditionnelles avec quatre jeux éducatifs numériques. Les résultats ont montré que les enfants commettaient souvent des erreurs courantes, telles que : laisser des espaces entre les unités non standard lors de la mesure, placer incorrectement les règles, et mal comprendre les intervalles numériques sur les règles. Ces résultats soulignent la nécessité pour les enseignants de corriger de telles erreurs afin d'améliorer la compréhension des mesures par les enfants. L'étude offre des informations précieuses sur les difficultés rencontrées par les enfants lorsqu'ils apprennent la mesure. Il fournit également des recommandations pratiques pour les enseignants et les décideurs afin de créer des stratégies et interventions pédagogiques efficaces.

MOTS CLÉS
École maternelle, géométrie, comparaisons, normes, mesures non standard, jeux éducatifs numériques

INTRODUCTION

The increasing focus on teaching mathematics in preschool highlights its importance in early education. Mathematics supports various cognitive domains, such as reading and science, and significantly impacts children's academic success. However, many students perceive it as one of the most challenging subjects (Fleming, 2019), leading to misconceptions. These misconceptions often hinder understanding fundamental concepts (Sophian, 2004), so identifying and studying these errors is essential for designing effective teaching interventions for mathematics.

Measurement is a critical area in the mathematics curriculum worldwide from prekindergarten to high school due to its practicality and widespread application in various aspects of daily life (National Council of Teachers of Mathematics, 2000). It emphasizes the relationships within mathematics as well as its connections to other fields, such as social studies, science, art, and physical education (Clements & Sarama, 2014). Although is highlighted the importance of measurement for teaching mathematics from early years, most research in preschool mathematics focuses on numbers and operations, while studies on measurement are quite limited (Kolovou et al., 2021; Petropoulou et al., 2024). Recent studies have focused on the effectiveness of teaching measurement concepts using traditional methods (Kotsopoulos et al., 2015; Matsuo & Nakawa, 2019; Van de Heuvel-Penhuizen & Elia, 2011) and digital games, which have been shown to improve learning in mathematics and other subjects (Aladé et al., 2016; Schenke et al., 2019). However, there is a lack of studies examining children's measurement errors. This research aims to fill that gap by exploring errors related to length measurement in preschool education.

This study is expected to provide valuable insights for various stakeholders. It will benefit teachers by enhancing their understanding of length measurement errors and support education consultants in planning relevant training seminars.

LITERATURE REVIEW

Measurement concepts include length, area, volume, weight, and time, with this study focusing specifically on length. According to Clements and Sarama (2014), length measurement involves assigning a number to an object's attribute, such as its length, relative to a unit. Key principles of length measurement include:

- Equal Partitioning: Dividing an object into equal parts (Clements & Sarama, 2014).

- Iteration of the Unit: Repeatedly using a smaller unit to measure a larger object (Kamii & Clark, 1997).
- Transitivity: Understanding that if object A equals object B and object B equals object C, then object A equals object C (Clements & Stephan, 2004).
- Conservation: Recognizing that an object's length remains the same even when moved (Wheatley, 1972).
- Distance Accumulation: Associating numerical words with the distance covered during unit iteration (Clements & Stephan, 2004).
- Relation to Number: Connecting counting, typically used for discrete quantities, to continuous quantities in measurement (Sarama & Clements, 2009).

Length measurement can be done in two ways: direct and indirect measurement, using either conventional (standard) tools, such as rulers, or nonconventional (nonstandard) tools, such as paperclips (Szilágyi et al., 2013). Direct measurement is typically introduced first in preschool curricula. It involves comparing, sometimes visually, objects and answering questions like "Which is bigger?" or "Which is taller?" Indirect measurement is a more complex process, where preschoolers often begin with nonconventional tools, such as pencils, paperclips, or even parts of their bodies like their feet, to measure objects. Eventually, they move on to using conventional tools, such as rulers or measuring tapes.

Preschool children frequently encounter difficulties when learning length measurement principles, leading to several common errors. During direct measurement, children often focus on one side of the object they are measuring, which can result in incorrect conclusions (Clements & Sarama, 2014). For instance, while children may understand that two parallel strips of the same length are equal if one strip is bent or reshaped, they might no longer recognize the lengths as equal (Carpenter & Lewis, 1976). Furthermore, children can efficiently compare the lengths of two objects but struggle when asked to compare three or more (Clarke et al., 2006; Williams & Shuard, 1982).

In indirect measurement with nonconventional tools, errors often arise when children leave gaps between the tools or overlap them while measuring the length of an object (Clements & Sarama, 2014; Lehrer, 2003). Another challenge is selecting only identical nonconventional tools for measurement. For example, children may use a combination of pencils and erasers, which leads to inaccurate results. This issue is also observed when children use conventional tools, as they might combine different units, such as meters and feet, to measure the same object (Clements & Sarama, 2014). When using conventional tools for indirect measurement, preschoolers face additional challenges. For instance, they often do not understand what the spaces between numbers on a ruler represent (Clements & Sarama, 2014; Clements & Stephan, 2004; Stephan et al., 2001). Children may also place the ruler incorrectly, such as starting at the number 1 instead of 0 or even placing the ruler upside down (Clements & Sarama, 2014; Gómez-Escobar et al., 2023; Nunes et al., 1993).

RESEARCH OBJECTIVES

The purpose of this paper is to identify the errors preschool children make in length measurement after participating in a teaching intervention that incorporates digital educational games (DEGs). The study investigates the types of errors children encounter during different measurement processes. Specifically, it examines:

1. the errors made by preschoolers when measuring directly,
2. the challenges they face when measuring indirectly with nonconventional units, and
3. the difficulties they encounter when measuring indirectly with conventional units.

METHODOLOGY

This study employs a qualitative methodology, specifically a case study approach, to explore preschool children's errors in length measurement. A case study allows for an in-depth investigation of individuals or groups using various methods of data collection, such as interviews (Creswell & Poth, 2016). The sample consists of 20 preschool children from the wider region of Achaia, whose parental consent had been obtained with the reassurance that anonymity would be respected.

The research included a teaching intervention that combined traditional methods with four digital educational games (DEGs), which previous studies have shown to be effective in enhancing learning. These games focused on length measurement, covering both direct and indirect methods using conventional and nonconventional units (see Appendix 1). The first game focused on direct measurement, requiring children to arrange similar objects in ascending or descending order based on their size. The second game also involved direct measurement, where children matched three objects to the appropriate train wagons according to their size. The third game addressed indirect measurement with nonconventional means and particularly dealt with the principal of unit iteration, where children used objects repeatedly to create a nonconventional ruler, to measure the length of a foot, and then selected the correct shoe from three options. The fourth game focused on indirect measurement with conventional tools, displaying objects parallel to a ruler on the screen, dealing with the principle of relation to number. Children were tasked with finding the length of the objects by observing the points on the ruler, aided by a dotted line that aligned with the object's edge.

The teaching intervention included activities based on the measurement principles of conservation and distance accumulation, while DEGs were mainly engaged in unit iteration and relation to number. In the conservation task, were given to the children three strings of the same size in a straight line and asked to compare their sizes and find the equals. Then, the shape of one string was changed and the children asked again about the equals, without touching or straightening them. As for the distance accumulation, children were asked to split into pairs, in which one should jump from a starting line as far as he could and the other should mark where he landed. After repeating it multiple times, children were asked to add all the distances of each jump to find the total distance covered and compare it with other pairs.

Data collection was conducted through semi-structured interviews to investigate preschoolers' errors. Two pictures, one of a shoe and one of a pen, were used during the interviews (see Appendix 2). For the shoe, children were first asked to choose between paper clips and sticks as the most suitable tools for measuring its length (direct comparison). Regardless of their choice, they were then asked to use paper clips to measure the shoe's length by placing them along its edge (indirect comparison with nonconventional means). For the pen, children were asked to measure its length using a ruler (indirect comparison with conventional means).

The children's measurement strategies were carefully recorded, organized, and analyzed with thematic analysis to identify and categorize the errors they made. The findings are presented in the results section.

RESULTS

The first research question explored the errors children made in measuring length directly. In this task, children were asked to choose between paper clips and sticks as the most suitable tool for measuring a shoe in the picture, using visual criteria. The correct choice was paper clips, as the sticks could not precisely measure the shoe's length. Table 1 presents the preschoolers'

choices following the teaching intervention, showing that most children correctly selected the paperclips over the sticks.

TABLE 1

Direct comparison of the nonconventional means

Non-conventional means	**Frequency**
Correct answers (paper clips)	13
Wrong answers (sticks)	7
Total	**20**

The second research question examined the errors children made when placing paper clips lengthwise along the shoe in the picture. These errors are summarized in Table 2. The most common error was leaving gaps between the paperclips, as illustrated in Figure 1. The second most frequent error involved placing the paperclips randomly on the paper, as shown in Figure 2, or positioning them only at the two edges of the shoe, as depicted in Figure 3.

TABLE 2

Indirect measurement with nonconventional means

Preschoolers' answers N	**Frequency**
Without errors	9
Placing paper clips randomly on the paper	1
Placing two paper clips at each side of the object	1
Placing with gaps	9
Total	**20**

FIGURES 1, 2, 3

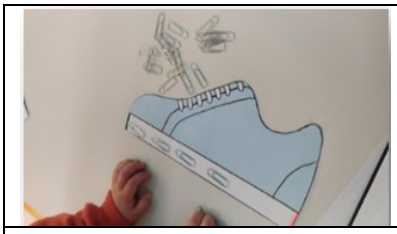


FIGURE 1
Placing paperclips with gaps

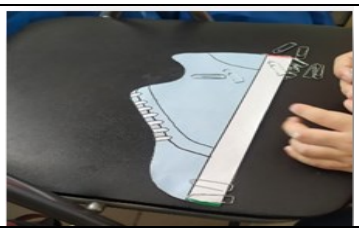


FIGURE 2
Paperclips placed randomly along the length of the object

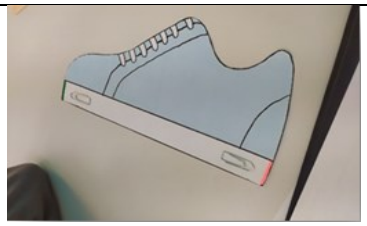


FIGURE 3
Paperclips placed only at the edges of the object

The third research question focused on how children measured the length of a pen using a ruler. Only one child correctly positioned the ruler, while several errors were observed during this task, as shown in Table 3.

TABLE 3
Indirect measurement of length with conventional means

Preschoolers' answers N	**Frequency**
Correct placement	1
No placement of the ruler on the paper	1
Placement of the ruler on the wrong starting number	6
Placement of the ruler's edge at the edge of the paper	4
Placement of the ruler's edge outside the paper	4
Vertical placement of the ruler	1
Upside-down placement of the ruler with its edge outside the paper	3
Total	**20**

For the task involving indirect measurement with conventional means, six different types of errors were identified. The most frequent error was placing the ruler on the wrong starting number for counting, as shown in Figure 4. Other errors included incorrectly placing the ruler at the edge of the paper (figure 5) or positioning it outside of the paper altogether (figure 6). In some cases, preschoolers made a combination of errors, such as placing the ruler upside down with its edge outside the paper (figure 7). Additional errors recorded included placing the ruler vertically (figure 8) or not placing it on the paper at all, instead pointing to the length of the object mentally with the ruler (figure 9).

FIGURES 4, 5, 6, 7, 8, 9

FIGURE 4
Ruler placed on the wrong starting number

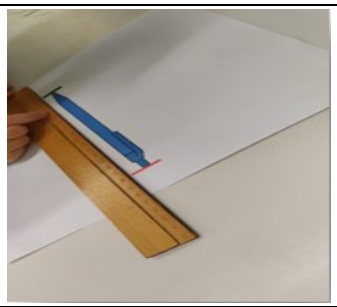


FIGURE 5
Ruler placed at the edge of the paper

FIGURE 6
Ruler's edge placed outside the paper

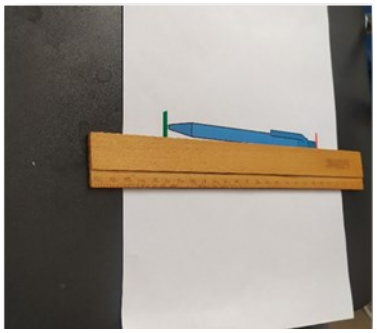


FIGURE 7
Ruler placed upside down with its edge outside the paper

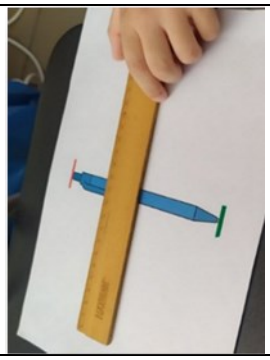


FIGURE 8
Ruler placed vertically

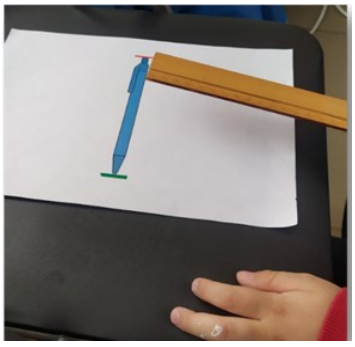


FIGURE 9
Mentally estimating the length with the ruler without placing it on the paper

DISCUSSION

This study investigated the errors preschool children make in three forms of length measurement: direct measurement, indirect measurement with nonconventional means, and indirect measurement with conventional means. The research began with a teaching intervention that combined traditional methods with digital educational games (DEGs). Afterward, a test was conducted following semi-structured interviews where children were asked to measure the length of two objects depicted in pictures using paperclips as nonconventional tools and a ruler as a conventional tool. The children's errors were recorded and analyzed.

Some of the observed errors are particularly noteworthy. Indirect measurement, children were asked to choose the most suitable nonconventional tool, using visual criteria, to measure the length of an object. No significant errors were noted here, except for the occasional incorrect choice of sticks, which were visually unsuitable for measuring the object's exact length. In indirect measurement with nonconventional means, more errors were observed. A common error, confirmed by prior research (Clements & Sarama, 2014; Lehrer, 2003), was leaving gaps between the paperclips when placing them lengthwise along the object. Other errors included placing paper clips randomly on the paper or only at the object's edges.

Indirect measurement with conventional means, errors frequently involved placing the ruler on a number other than zero to start measuring, an issue documented in existing literature (Clements & Sarama, 2014). Additional errors included placing the ruler outside the object's edges, positioning it upside down, or aligning it vertically with the object. These errors reflect difficulties with the proper use of a ruler.

While many of the errors align with findings from previous studies, some new ones emerged, especially during indirect measurement with both conventional and nonconventional means. New errors with nonconventional tools included placing paper clips only at the edges of the object or randomly along its length. With conventional tools, newly observed errors involved positioning the ruler vertically or attempting to mentally estimate the object's length without physically aligning the ruler.

Teachers can benefit greatly from this knowledge by focusing on eliminating these errors through targeted interventions. In order to reduce the errors observed during indirect measurements by conventional and nonconventional means, specific teaching practices can be implemented. First, teaching the correct positioning of measurement tools is important. Preschoolers can practice placing the ruler from the zero point on different objects, ensuring that it is correctly aligned with the edge of the object. This aids in avoiding errors resulting from incorrectly starting to measure. In addition, the development of alignment skills can be achieved through exercises where students align the ruler correctly, avoiding vertical or upside-down placements. At the same time, the use of nonconventional tools, such as paper clips or string, encourages practice in correct alignment and consistency in measurements. Observing and analyzing both correct and incorrect examples is another effective practice. Students can observe examples of correctly aligned tools and discuss why errors lead to inaccurate results. In addition, digital educational games involving alignment and measurement can reinforce these skills in a fun way. Furthermore, consultants designing training seminars for teachers can use this knowledge to improve educators' skills and introduce modern teaching tools.

There are several limitations to the present study that should be noted. The short duration of the research may have prevented the observation of more errors or the elimination of those recorded. Additionally, the small sample size means the findings cannot be generalized to a broader population. Some errors documented in existing literature may not have been observed here due to the positive impact of the teaching intervention. Another limitation was the inability to control external factors, such as stimuli children were exposed to outside the

school environment. Moreover, it was not always possible to determine whether the students' errors stemmed from this teaching approach or from other previous approaches to length measurement that may have contributed to cognitive barriers. Future research with larger samples or intervention's duration can provide more insights in this field. Moreover, research could explore length measurement further, along with other attributes like weight or volume, using larger samples and longer interventions. Comparing traditional teaching methods with digital game-based approaches could provide valuable insights into reducing errors. Finally, exploring children's errors to find the reasons why these cognitive obstacles were created all along could help educators eliminate them.

REFERENCES

Aladé, F., Lauricella, A. R., Beaudoin-Ryan, L., & Wartella, E. (2016). Measuring with Murray: Touchscreen technology and preschoolers' STEM learning. *Computers in Human Behavior*, *62*, 433-441.

Carpenter, T. P., & Lewis, R. (1976). The development of the concept of a standard unit of measure in young children. *Journal for Research in Mathematics Education*, *7*(1), 53-58.

Clarke, B., Cheeseman, J., & Clarke, D. (2006). The mathematical knowledge and understanding young children bring to school. *Mathematics Education Research Journal*, *18*(1), 78-102.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach*. Routledge.

Clements, D. H., & Stephan, M. (2004). Measurement in pre-K to grade 2 mathematics. In D. H. Clements & J. Samara (Eds), *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 299-317). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Creswell, J. W., & Poth, C. N. (2016). *Qualitative inquiry and research design: Choosing among five approaches*. Sage publications.

Fleming, G. (2019). Why Math is more difficult for some students. https://www.thoughtco.com/why-math-seems-more-difficult-for-some-students-1857216.

Gómez- Escobar, A., Rodrigues, M., & Fernández-Cézar, R. (2023). Children's understanding of length measurement using a ruler in preschool and primary education: A cross-national longitudinal study. *The Journal of Mathematical Behavior*, *69*, 101048. https://doi.org/10.1016/j.jmathb.2023.101048.

Kamii, C., & Clark, F. (1997). Measurement of length: The need for a better approach to teaching. *School Science and Mathematics*, *97*(3), 116-121.

Kolovou, S., Koutsolabrou, I., Lavidas, K., Komis, V., & Voulgari, I. (2021). Digital games in early childhood education: Greek preschool teachers' views. *Mediterranean Journal of Education*, *1*(2), 30-36. https://doi.org/10.26220/mje.3764.

Kotsopoulos, D., Makosz, S., Zambrzycka, J., & McCarthy, K. (2015). The effects of different pedagogical approaches on the learning of length measurement in kindergarten. *Early Childhood Education Journal*, *43*, 531-539.

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds), *A research companion to principles and standards for school mathematics* (pp. 179-191). Reston, Virginia: NCTM.

Matsuo, N., & Nakawa, N. (2019). Preschool children's understanding of length and area measurement in Japan. In *Eleventh Congress of the European Society for Research in Mathematics Education (CERME11)*, No. 17. Freudenthal Group; Freudenthal Institute; ERME.

National Council of Teachers of Mathematics. (2000). Principles and standards for school Mathematics, NCTM: Reston, VA, USA.

Nunes, T., Light, P., & Mason, J. (1993). Tools for thought: The measurement of length and area. *Learning and Instruction*, *3*(1), 39-54.

Petropoulou, A., Lavidas, K., & Papadakis, S. (2024). Investigation of preschoolers' mathematical skills: A systematic literature review. *Educational Process. International Journal*, *13*(2), 31-51. https://doi.org/10.22521/edupij.2024.132.3.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research. Learning trajectories for young children*. London, New York: Routledge.

Schenke, K., Redman, E. J., Chung, G. K., Chang, S. M., Feng, T., Parks, C. B., & Roberts, J. D. (2020). Does "measure up!" measure up? Evaluation of an iPad app to teach preschoolers measurement concepts. *Computers & Education*, *146*, 103749. https://doi.org/10.1016/j.compedu.2019.103749.

Sophian, C. (2004). Mathematics for the future: Developing a Head Start curriculum to support mathematics learning. *Early Childhood Research Quarterly*, *19*(1), 59-81.

Stephan, M., Cobb, P., Gravemeijer, K., & Estes, B. (2001). The role of tools in supporting students' development of measuring conceptions. In A. A. Cuoco (Ed.), *The roles of representation in school mathematics. 2001 yearbook* (pp. 63-76). National Council of Teachers of Mathematics.

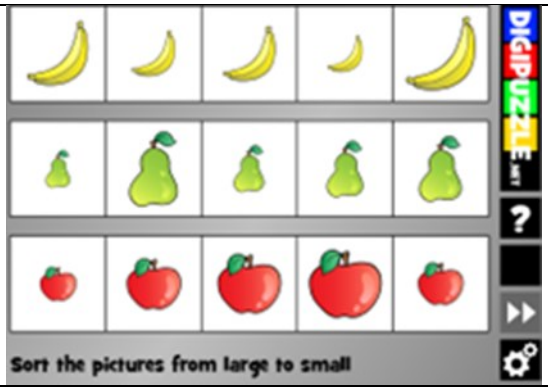
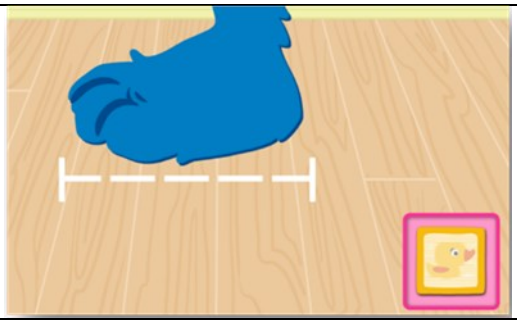
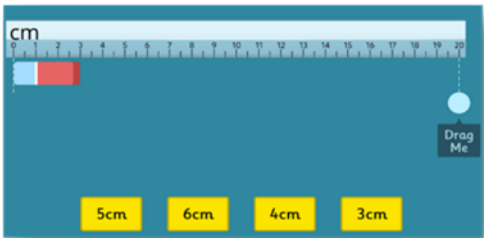
Szilágyi, J., Clements, D. H., & Sarama, J. (2013). Young children's understandings of length measurement: Evaluating a learning trajectory. *Journal for Research in Mathematics Education*, *44*(3), 581-620. https://doi.org/10.5951/jresematheduc.44.3.0581.

Van den Heuvel-Panhuizen, M., & Elia, I. (2011). Kindergartners' performance in length measurement and the effect of picture book reading. *ZDM*, *43*, 621-635.

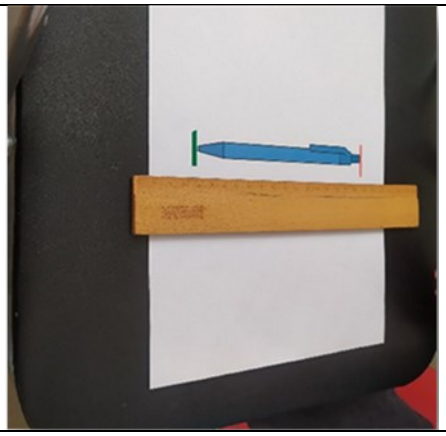
Wheatley, G. H. (1972). *The Piagetian conservation of length as effected by two alternate modes of spatial translation: A methodological problem*. Final Report. https://files.eric.ed.gov/fulltext/ED078087.pdf

Williams, E., & Shuard, H. (1982). *Primary Mathematics today*. Essex: Longman.

APPENDIX 1

	
1st DEG: Sorting Objects by Size, Length or Position (https://www.cokogames.com/order-by-size/play/)	2nd DEG: Size sorting (https://www.cokogames.com/size-sorting/play/)
	
3rd DEG: Measure that foot (https://sesameworkshop.org/resources/measure-that-foot/)	4th DEG: Measuring in cm (https://www.topmarks.co.uk/maths-games/measuring-in-cm)

APPENDIX 2

	
Indirect comparison with nonconventional means (paper clips & sticks)	Indirect comparison with conventional means (ruler)